Kai-Uwe Schmidt

On Spectrally-Bounded Codes for Multicarrier Communications

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On Spectrally-Bounded Codes for Multicarrier Communications

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Kai-Uwe Schmidt
Dresden, April 2007

To Leonard,

who was born just before the completion of this thesis.

Kurzfassung

Das Sendesignal in einem Mehrträgersystem entsteht durch eine orthogonale Transformation eines Datenworts. Es folgt daraus, dass die maximale Momentanleistung des Sendesignals um ein Vielfaches höher sein kann als die mittlere Sendeleistung. Dies stellt einen hohen Anspruch an die Implementierung des Systems, was als entscheidender Nachteil von Mehrträgersystemen betrachtet wird. Das Verhältnis von maximaler und mittlerer Sendeleistung (der Spitzenwert) lässt sich allerdings durch gezielte Ausnutzung der Redundanz eines Fehlerschutzcodes reduzieren. Die vorliegende Dissertation beschäftigt sich mit der Konstruktion solcher Codes für zwei praxisrelevante Mehrträgersysteme: Orthogonales Frequenzmultiplex (OFDM) und Multicode Codemultiplex (MC-CDMA).

Der erste Teil der Arbeit beinhaltet eine Analyse der maximalen Spitzenwertreduktion in einem OFDM System, die durch Drehung der Koordinaten eines bekannten binären Codes erreicht werden kann. Anhand der erzielten Ergebnisse wird gezeigt, dass aus der Literatur bekannte suboptimale Konzepte zur Bestimmung der Drehwinkel den Spitzenwert oftmals bis nahe an die theoretische Grenze reduzieren.

In einem weiteren Teil werden unter Benutzung der spektralen Eigenschaften von komplementären Paaren und deren Verallgemeinerungen, wie fast-komplementäre Paare und komplementäre Mengen, obere und untere Schranken für den maximalen Spitzenwert von bestimmten Nebenklassen eines verallgemeinerten Reed–Muller Codes erster Ordnung hergeleitet. Durch Vereinigungen solcher Nebenklassen lassen sich Codes für OFDM und MC-CDMA mit Fehlerkorrektureigenschaften und strikt begrenztem Spitzenwert konstruieren. Oftmals erhält man dabei Codes, deren Parameter besser sind als die bisher bekannter Codes. Ferner liefern die erzielten Ergebnisse theoretische Erklärungen für einige Vermutungen und experimentelle Beobachtungen in der Literatur.

Schließlich werden so genannte verallgemeinerte Bent-Funktionen in Verbindung mit algebraischen Codes über $\mathbb{Z}_4$ (verallgemeinerte Reed–Muller, Kerdock und Delsarte–Goethals Codes) genutzt, um nichtbinäre Fehlerschutzcodes für MC-CDMA zu konstruieren. Diese Codes reduzieren den Spitzenwert auf den kleinstmöglichen Wert und ergänzen vergleichbare bekannte binäre Codes.

Abstract

In multicarrier communications the transmitted signal is obtained by applying an orthogonal transform to a block of data symbols. As a consequence, the peak power of the transmitted signal can be much larger than its mean power, which makes the practical implementation of the system a challenging task. The redundancy of carefully designed error-correcting codes can be exploited to control the ratio of the peak and the mean power of the transmitted signal. In this thesis such codes are studied for two types of multicarrier communications systems: orthogonal frequency-division multiplexing (OFDM) and multicode code-division multiple access (MC-CDMA).

In a first part of this thesis a well-known approach for the construction of such codes for OFDM is examined. This concept involves a rotation of the coordinates of a known binary code by phase shifts that are independent of the individual codewords. Bounds for the maximum reduction of the peak-to-mean power ratio are proved, and it is shown that most phase-shift designs in the literature, obtained using suboptimal optimization methods, produce peak-to-mean power ratio reductions that are close to the theoretical limit.

The spectral properties of complementary pairs and generalizations, namely near-complementary pairs and complementary sets, are then exploited to prove upper and lower bounds on the peak-to-mean power ratio of certain cosets of a first-order generalized Reed–Muller code. By taking unions of such cosets inside higher-order generalized Reed–Muller codes, error-correcting codes for OFDM and MC-CDMA with strictly bounded peak-to-mean power ratios are constructed. These codes complement and, in many situations, improve existing coding schemes. Moreover these results provide theoretical support for conjectures and empirical observations in the literature.

Finally generalized bent functions in connection with algebraic codes over $\mathbb{Z}_4$, namely generalized Reed–Muller, Kerdock, and Delsarte–Goethals codes, are exploited to construct families of so-called constant-amplitude error-correcting codes for MC-CDMA, which are codes that reduce the peak-to-mean power ratio to the least possible value. These codes complement previously proposed binary constant-amplitude codes.

Contents

Chapter 1

Introduction

1.1 Motivation

Orthogonal frequency-division multiplexing (OFDM) represents a key concept in the development of wired and wireless communications systems in the past decade [2]. It provides excellent ability to cope with multipath propagation and fast-moving environment. OFDM has been proposed and standardized for many wireless applications such as wireless local area networks (WLAN), digital audio and video broadcasting (DAB/DVB), as well as for wired applications such as in digital subscriber line (DSL) systems.

Code-division multiple access (CDMA) has been standardized for third generation (3G) mobile telephone systems such as the universal mobile telecommunication system (UMTS) and CDMA2000. Multicode CDMA (MC-CDMA) is a simple concept, which is backwards-compatible to conventional CDMA. It was proposed in order to satisfy the temporary demand for higher data rates of individual users in a CDMA system [45].

Both concepts, OFDM as well as MC-CDMA, are special cases of multicarrier communications systems. The latter is a technique for data transmission where the transmitted data is divided into blocks (codewords), which are used to modulate a set of pairwise orthogonal waveforms (carriers). The sum of these modulated carriers is the transmitted signal. Mathematically, this signal is obtained by applying an orthogonal transform to the transmitted codeword, so the transmitted signal may be viewed as the spectrum of the codeword. Therefore, at the receiving side, the data can be recovered by applying the corresponding inverse transform. In OFDM the transform is the Fourier transform. MC-CDMA essentially employs a Hadamard transform.

The principal disadvantage of all kinds of multicarrier systems is that some codewords exhibit a high peak-to-mean power ratio (we shall use the terms *peak-to-mean envelope power ratio* (PMEPR) and *peak-to-average power ratio* (PAPR) in OFDM and in MC-CDMA, respectively). That is, the peak transmit power can be many times the average transmit power, which happens when the modulated carriers line up at some time instant. In other words, high peaks occur in the spectrum (with respect to the transform employed in the system) of certain codewords.

Thus, in order to ensure a distortionless transmission, all components in the transmission chain must be linear across a wide range of signal levels. This makes the transmitter considerably more expensive than that in a single-carrier system. Moreover, most of the time, the components in the transmitter are operated at levels much below their maximum input level, which results in power inefficiency. The latter issue is particularly acute in mobile applications, where battery lifetime is a major concern. On the other hand, nonlinearities in the transmission chain may lead to a loss of orthogonality among the carriers and to out-of-band radiation. The former has the effect of degrading the total system performance and the latter is subject to strong regulations. Clipping and filtering of the signal before digital-to-analog conversion has been shown to alleviate the problem [55], however, performance degradation and out-of-band radiation (or at least one of them) remains an issue.

This provides clear motivation to reduce the peak-to-mean power ratio in multicarrier systems systematically. This power-control problem can be solved by introducing redundancy such that the transmission of codewords with high peak-to-mean power ratios is avoided. Thus the problem is shifted to identifying those unwanted codewords.

Probabilistic methods have received a great amount of attention. Such an approach involves a multiple representation of the transmitted information. An algorithm then selects a favorable representation, which is used as the transmitted codeword. Popular variants of this technique are *selected mapping* and *partial transmit sequences* [66]. While in most situations the peak-to-mean power ratio is reduced, the main drawback of this method is the requirement of complex on-line calculations in the transmitter. Moreover the transmission of side information is usually necessary.

A more sophisticated technique was introduced in [50] and further developed in [117]. The idea here is to draw the codewords from a special block code, which is designed off-line such that the peak-to-mean power ratio of all codewords is not exceeding a certain threshold. This technique becomes particularly elegant if the inherent redundancy can be exploited for error-correction [48]. Thus this approach combines error-correction coding and the reduction of the peak-to-mean power ratio in multicarrier systems. The goal of this thesis is the analysis and construction of such codes for two popular multicarrier systems: OFDM and MC-CDMA.

1.2 Related Work

The problem statements of constructing error-correcting codes with low peak-to-mean power ratio in [50], [117], and [48] has initiated a great deal of research. Most of the research was focused on codes for OFDM, some on codes for MC-CDMA. There exists a number of approaches to design such codes. In what follows we review them briefly.

A simple approach, proposed by Jones and Wilkinson in [48], for designing OFDM codes with low PMEPR is to offset a well-understood code. That means the coordinates of a given code are rotated by fixed phase shifts, which are selected such that the maximum PMEPR, taken over all codewords, is minimized. Based on learning techniques, an algorithm to find such phase shifts was presented in [48]. This algorithm works, however, only for very short codes. The approach became much more attractive when Tarokh and Jafarkhani [106] proposed a computationally feasible algorithm to find good phase shifts, provided that an efficient maximum-likelihood decoding algorithm for the original code is known. Some complexity reductions of this algorithm have been reported by Wunder and Boche [119]. However there is no guarantee that the algorithm finds the optimum phase shifts, neither does this approach imply anything about the achievable PMEPR reduction.

Paterson and Tarokh [83] proposed to use so-called trace codes in OFDM. These are linear codes and include the duals of BCH codes, the quaternary versions of Kerdock and Delsarte–Goethals codes, as well as the weighted-degree trace code (which is the most general trace code). Using bounds on hybrid character sums, it was shown that, when the zero codeword is excluded, the PMEPR of trace codes is of the order $(\log n)^2$, where n is the length of the code. In [102] Solé and Zinoviev provided a slightly more general treatment of this issue and corrected some minor mistakes in [83]. Although the PMEPR of these codes is rather high for practical n, asymptotically the PMEPR is much lower than n, which is the worst-case PMEPR for uncoded transmission.

A number of contributions [117], [111], [72] proposed to use Golay sequences [30] in OFDM. A Golay sequence is a member of a Golay complementary pair, which has the property that the aperiodic autocorrelation sidelobes of the two sequences in the pair sum to zero. This property implies that the PMEPR of any Golay sequence is at most 2 [84]. The work on Golay sequences for OFDM culminated in the paper by Davis and Jedwab [22], which revealed a striking connection between Golay sequences and Reed–Muller codes. More specifically, it was shown that a family of binary Golay sequences of length 2^m organizes in $m!/2$ cosets of $\mathrm{RM}_2(1,m)$ inside $\mathrm{RM}_2(2,m)$, where $\mathrm{RM}_2(r,m)$ is the Reed–Muller code of order r and length 2^m [60, Chapter 13]. Similarly for $h > 1$ [22] identifies $m!/2$ cosets of $\mathrm{RM}_{2^h}(1,m)$ comprised of polyphase Golay sequences inside $\mathrm{ZRM}_{2^h}(2,m)$. Here $\mathrm{RM}_q(r,m)$ and $\mathrm{ZRM}_q(r,m)$ are generalizations of the classical Reed–Muller code over q-ary alphabets. Consequently [22] suggests OFDM coding schemes for 2^h-ary phase-shift keying constellations with large minimum distance and PMEPR bounded by 2.

The major drawback of this result is that the rate of the codes rapidly tends to zero as the number of carriers increases. Therefore Davis and Jedwab [22] proposed to include further second-order cosets of $\mathrm{RM}_{2^h}(1,m)$. This was motivated by the observation that second-order cosets of $\mathrm{RM}_{2^h}(1,m)$ seem to organize naturally in classes with the same PMEPR upper bound. For 16 carriers [22] contains a computationally obtained ranking of second-order cosets according to their PMEPR, which suggests trade-offs between the code rate and the PMEPR without compromising the minimum distance. However the complexity of such a coset ranking becomes unacceptably high for larger number of carriers.

Further theoretical advance was made by Paterson [79] by showing that *any* coset of $\mathrm{RM}_q(1,m)$ inside $\mathrm{RM}_q(2,m)$ can be decomposed into complementary sets [110] of the same size. By generalizing the argument in [84], Paterson [79] established that any sequence lying in a complementary set of size N has PMEPR at most N, which yields an upper bound on the PMEPR of any coset of $\mathrm{RM}_q(1,m)$ inside $\mathrm{RM}_q(2,m)$. This explains much, though not all, of the empirical observations in [22]. The results in [79] led to OFDM coding schemes with PMEPR bounded by 2^k, where k is a positive integer. The connection of cosets of $\mathrm{RM}_q(1,m)$ and complementary sets inspired further studies in [105], [78], and [16], however, without explicitly deriving OFDM coding schemes.

Up to now the construction of codes for MC-CDMA with low PAPR was limited to binary codes. Research was particularly focused on the construction of constant-amplitude codes, which are codes that reduce the PAPR to least possible value 1.

Wada *et al.* [114] constructed the largest possible binary constant-amplitude codes of lengths 4 and 16. The minimum distance of these codes was then analyzed by the same authors in [115]. Ottoson [74] proposed a simple algorithm to identify codewords with high PAPR, which are removed from the set of binary n-tuples to obtain a code of length n with low PAPR. A second algorithm then removes further codewords from this code such that the additional redundancy can be exploited for error correction. However this scheme is only useful for small n.

A coding-theoretic framework for binary codes in MC-CDMA has been established by Paterson in [81]. It was shown that codewords with low PAPR are exactly those words that are far from $\mathrm{RM}_2(1, m)$, the first-order Reed–Muller code. Based on this fact, fundamental trade-offs between PAPR, minimum distance, and code rate of binary codes have been proved in [81]. Moreover it was shown that codewords that are farthest away from $\mathrm{RM}_2(1, m)$ (words corresponding to bent functions [87], [60, Chapter 14]) have PAPR equal to 1. Though first recognized by Wada [113], this connection has been exploited in [81] to construct several families of binary constant-amplitude codes of length 2^{2m}, where m is a positive integer. These codes are unions of cosets of $\mathrm{RM}_2(1, 2m)$ inside higher-order Reed–Muller, Kerdock, or Delsarte–Goethals codes [60, Chapter 15]. Therefore they simultaneously enjoy the lowest possible PAPR and high minimum distance. Reference [81] also proposes several code families with PAPR greater than 1, particularly for lengths 2^{2m+1} (m is a positive integer) for which binary constant-amplitude codes cannot exist.

Recently, Solé and Zinoviev [103] constructed binary codes with PAPR much greater than 1. However in many situations their parameters beat the Gilbert–Varshamov-style lower bound derived in [81].

1.3 Contribution and Outline

In the next chapter we will introduce some useful notation, present preliminary results, and state the power-control problems in OFDM and MC-CDMA systems explicitly.

Chapter 3 contains an analysis of the PMEPR of phase-shifted codes. Motivated by previous designs of phase shifts in [48], [106], and [119] using suboptimal methods, we ask: given a binary code, how much PMEPR reduction can be obtained when the phase shifts are taken from a 2^h-ary phase-shift keying constellation? We will establish a lower bound on the PMEPR, which is related to the covering radius of the binary code. The general case, where the phase shifts can be arbitrary, is then recovered by analyzing this bound for $h \to \infty$. Informally, our bound states that the smaller the covering radius of the binary code, the less PMEPR reduction is possible. We will then apply the bound to some well understood codes, including BCH codes and their duals, Reed–Muller codes, and convolutional codes, and compare the results with some phase-shift designs from the literature. The results of Chapter 3 are accepted for publication in [93].

The main contribution of Chapter 4 is a construction of cosets of $\mathrm{RM}_q(1,m)$ comprising sequence pairs that exhibit a near-complementary property. We prove an upper bound on the PMEPR of these cosets and establish the tightness of this bound in certain cases. As a corollary we recover the cosets from [22] that contain Golay complementary pairs. More importantly, we identify many new classes of cosets of $\mathrm{RM}_q(1,m)$ with low PMEPR, showing that several previously unexplained phenomena, observed, e.g., in [22], can now be understood as part of a general framework. These results lead to a number of OFDM coding schemes whose PMEPRs are bounded by values between 2 and 4. We also study maximum-likelihood decoding algorithms for $\mathrm{RM}_q(1,m)$, which can be used as well to perform maximum-likelihood decoding for any union of cosets of $\mathrm{RM}_q(1,m)$. The results on the construction of cosets of $\mathrm{RM}_q(1,m)$ are published in [90], special cases are also contained in [95]. The material about maximum-likelihood decoding algorithms is published in [94] and [89].

In Chapter 5 we continue the study of cosets of $\mathrm{RM}_q(1,m)$ by relating such cosets with complementary sets. This chapter essentially contains generalizations of the results obtained by Paterson in [79]. We show that any coset of $\mathrm{RM}_q(1,m)$ comprises sequences lying in complementary sets of size 2^k, where k is a positive integer that can be easily determined from the generalized Boolean function associated with a word in the coset. Consequently we obtain an upper bound on the PMEPR of any coset of $\mathrm{RM}_q(1,m)$. Although the bound tends to be loose for large k, the bound is shown to be tight in many situations, especially when $k \in \{1,2\}$. We then combine these results with nonbinary generalizations of the Reed–Muller code to construct OFDM coding schemes with low PMEPR. When directly possible, we compare our codes with existing schemes in [79], and show that in most situations our codes are improvements. The key results of this chapter are published in [92].

In Chapter 6 we construct codes for MC-CDMA, where we focus especially on practically important nonbinary codes. We prove that the PAPR of any coset of $\mathrm{RM}_q(1,m)$ is at most its PMEPR, which implies that the OFDM coding schemes developed in Chapters 4 and 5 are also suited to bound the PAPR in MC-CDMA systems. We also show that it is impossible to use this approach to construct constant-amplitude codes when $q \equiv 0 \pmod 4$, which includes the practical cases of 2^h-ary phase-shift keying modulation when $h > 1$. We will then prove an upper bound for the PAPR of appropriately expurgated trace codes, in particular of the weighted-degree trace code, which has been previously proposed to limit the PMEPR in OFDM systems. Another principal goal of Chapter 6 is the construction of quaternary constant-amplitude codes. It is shown that such a code must be comprised of words corresponding to a kind of generalized bent functions. We provide a number of constructions of such functions and connect them with algebraic codes over $\mathbb{Z}_4$, in particular with the quaternary Reed–Muller codes $\mathrm{RM}_4(r,m)$ and $\mathrm{ZRM}_4(r,m)$ as well as with the quaternary Kerdock and Delsarte–Goethals codes, proposed in [36]. These coding schemes may be viewed as quaternary analogs of the binary codes developed in [81]. We also present mappings from binary to quaternary constant-amplitude codes. The results on quaternary constant-amplitude codes are submitted for publication in [91].

Chapter 7 contains a summary of the results in this thesis.