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**Jens Wagner**

**Design of a Dual Band RF Beamsteering  
Frontend for Frequency Modulated Continuous  
Wave Radar Systems**

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DESIGN OF A DUAL BAND RF BEAMSTEERING  
FRONTEND FOR FREQUENCY MODULATED CONTINUOUS  
WAVE RADAR SYSTEMS

Jens Wagner

von der Fakultät Elektrotechnik und Informationstechnik  
der Technischen Universität Dresden  
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## Zusammenfassung

Die vorliegende Arbeit präsentiert den Entwurf eines Zweiband-Hochfrequenz-*Beamsteering-Frontends* für frequency modulated continuous wave (FMCW)-Radarsysteme. Das entworfene Frontend unterstützt sowohl das 2.4 als auch das 5.8 GHz Industrial, Scientific, Medical (ISM)-Band.

Es existieren gute Arbeiten sowohl zu Zweiband-FMCW-Radaren und digitalen *Beamsteering*-Systemen. In dieser Arbeit soll die Kombination der Signale im Hochfrequenzbereich realisiert werden. Es sollen sowohl die Vorteile von Frequenz- als auch von Antennendiversität genutzt werden können.

Es wird zunächst eine Signaltheorie für FMCW-Radare in Szenarien mit Mehrwegeausbreitung formuliert. Daraus werden Anforderungen an das Mehrantennen-*Frontend* abgeleitet. Es werden verbesserte Amplitudenstellglieder entwickelt, modelliert und charakterisiert. Ein verbesserter Vektormodulator wird als Baustein für ein Vier-Antennen-*Array* entwickelt und hergestellt. Das Zweiband-Hochfrequenz-*Beamsteering-Frontend* wurde als integrierter Schaltkreis entworfen und gefertigt. Ein System, bestehend aus dem *Frontend* und einem existierenden FMCW-Radar, wurde auf einer Leiterplatte entworfen und realisiert. Dieses System wurde in verschiedenen Szenarien unter zum Teil deutlichen Mehrwegeausbreitungsbedingungen sowie unter Bedingungen ohne Sichtverbindung (non line-of-sight - nLOS) charakterisiert.

Der entworfene Vektormodulator erlaubt einen Phasenstellbereich von  $360^\circ$  und einen Amplitudenstellbereich von mehr als 40 dB. Dabei beträgt der root mean square (RMS) Phasenfehler lediglich  $6^\circ$ , der RMS Amplitudenfehler 0.16 dB. Der Vektormodulator hat einen Leistungsverbrauch von 87 mW und benötigt  $0.2 \text{ mm}^2$  Chipfläche.

Die Messungen des Radarsystems zeigen, dass der Distanzfehler durch Verwendung eines Hochfrequenz-*Beamsteering-Frontends* deutlich gesenkt werden kann. Es wurde erstmals ein Abstandsmesssystem basierend auf Radiowellen unter nLOS Bedingungen charakterisiert. Es hat dabei seine Vorteile gegenüber Ein-Antennen-*Frontends* demonstriert. Die mittlere Messabweichung des Abstandes konnte sowohl im 2.4 als auch im 5.8 GHz Band von etwa einem Meter bei der Nutzung von nur einer Antenne auf ca. 30 cm unter Verwendung des *Beamsteering frontends* gesenkt werden, dies entspricht einer Reduktion der Messabweichung auf etwa ein Drittel. Die Standardabweichung wurde im Schnitt sogar um Faktor vier verringert. Das realisierte *Beamsteering*-Radarsystem verbraucht im Betrieb ca. 270 mA aus einer 12 V Quelle. Es erlaubt einen Zweibandbetrieb und ermöglicht so die gleichzeitige Nutzung von Frequenz- und Antennendiversität.





## Abstract

In this work, the development of a dual band radio frequency (RF) beamsteering frontend for a frequency modulated continuous wave (FMCW) radar system is presented. The frontend supports both the 2.4 and the 5.8 GHz Industrial, Scientific, Medical (ISM) bands.

There have been great works on both dual band FMCW radars and digital beamsteering systems. In this work, the antenna signals will be combined in the RF domain while at the same time combining the benefits from spatial and frequency diversity receivers.

A signal theory on FMCW radar systems in multipath conditions is formulated. From this theory, requirements for the RF beamsteering frontend are derived. Improved amplitude control stages were developed, modeled and characterized. An improved vector modulating circuit as a building block for a four antenna array has been designed and measured. The dual band RF frontend application specific integrated circuit (ASIC) was designed and fabricated. A system has been developed on a printed circuit board (PCB) combining the designed frontend with an existing FMCW radar system. The system was characterized in different scenarios with heavy multipath effects and even in non line-of-sight (nLOS) conditions.

The designed vector modulator features a phase control range of  $360^\circ$  and more than 40 dB of amplitude control range. At the same time, the root mean square (RMS) phase and amplitude error are  $6^\circ$  and 0.16 dB, respectively. The vector modulator shows a power consumption of 87 mW and requires  $0.2 \text{ mm}^2$  of chip area.

The measurements show that the distance error could be reduced considerably using an RF beamsteering frontend. For the first time, a radio wave distance measurement system was benchmarked in nLOS conditions, showing its benefits compared to single antenna frontends. The mean distance measurement error was reduced from around one meter using only one antenna to around 30 cm using the beamsteering frontend, which means that the distance measurement error was reduced to around one third. The standard deviation of the distance measurement error could even be decreased by a factor of four. The realized beamsteering radar system requires 270 mA from a 12 V supply. It allows for dual band signal processing and thus enables to exploit spatial diversity and frequency diversity at the same time.



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# 1 Introduction

## 1.1 Historical and Ethical Aspects

The history of navigation is closely linked to the development of seamanship and goes back in time as far as several thousand years BC. The Minoans of Crete, a civilization flourishing from 3650 to 1400 BC, were among the first people using celestial navigation instead of sailing along the shores. The earliest Nautical charts date from the sixth century BC. Throughout history, several inventions have been changing the art of navigation continuously. Several devices to measure altitudes and latitudes of stars have been developed in all parts of the world. The invention of the compass in the 11th century allowed navigation when stars were not visible, a piece of iceland spar was possibly the famous sunstone that allowed the vikings to determine the suns position on an overcast sky.

A systematization of the sun's declination, winds, currents and empirical observations from seafaring allowed Portugal in the early 16th century to establish ocean routes all over the world. The devices used to measure altitude and latitude were improved. In 1578, a patent was granted for a device which allowed to measure the ship's speed. While accurate timing is crucial for precise navigation, hourglasses were still in use by the Royal Navy of Britain until 1839. With the quadrant invented in 1631, inclinations of objects could be measured with an accuracy of one minute of arc. The first sextant was invented in 1757, allowing mariners to determine their longitude accurately. Beginning in the 1890s, the first radios found use in ships, by 1904, navigators were able to check their chronometers using wireless signals. The first radiobeacons were installed in 1921. In 1937, the first radar prototype was demonstrated on a ship and in 1942, the first regional navigation system LORAN started to operate.

As of 1995, the American global positioning system (GPS) is officially operational. At the time, this was the first global positioning system, around 30 satellites are used as reference stations. Since then, three additional global navigation satellite systems (GNSS) were built: GLONASS of the Russian Federation, GALILEO of the European Union and Beidou of China. Today, with receivers in every mobile phone, it is fair to say that GNSS have changed our perception of navigation completely.

However, several challenges still remain unsolved. The low signal level of GPS and its limited accuracy make it impossible to use in indoor applications. Over the years, this motivated the development of several systems for local positioning us-

ing radio waves as well as GNSS augmentation systems like assisted GPS (A-GPS).

Among the various systems using radio waves for localization purposes, several physical principles can be used for the distance measurement: received signal strength indication (RSSI) based systems, ultra wide band (UWB) or pulse-based approaches ([Ubi16], [BeS16]), phase measurements ([Zig16]) and frequency modulated continuous wave (FMCW) radars ([Jor15], [Gie10]) are the most commonly used. With these systems, the performance of wireless location systems has improved tremendously, now allowing positioning in indoor environments with an error below one meter. The systems are already quite mature and begin to penetrate the consumer market, the first development kits with smartphones containing dedicated positioning frontends are available by now ([BeS16]).

Although wireless positioning has advanced a lot during the last decades, the indoor reflective channel is still a tremendous challenge and subject to active research. Especially non line-of-sight (nLOS) conditions increase positioning errors and are usually faced by placing more reference stations within the infrastructure. One promising measure to reduce errors in these surroundings is the use of multiple antennas. Combining the signals from these antennas in the radio frequency (RF) domain (RF beamsteering) will reduce system complexity and size considerably.

Despite all the conveniences positioning systems have brought to our daily lives, we must never forget that many of the developed positioning systems (GPS and GLONASS are only two examples) were initiated by the military. Many of the boundary conditions of GPS can be attributed directly to this fact. Only the US military can benefit from the full accuracy the system offers. GPS is used to steer rockets and drones, it is not a civilian project. In the European Union, several projects are funded aiming at localizing boats with refugees in the Mediterranean Sea. The goal is to be able to spot these boats before they reach European waters and prevent them from entering the continent (see [LOB16] as an example). The act of determining the position of people without or against their own will is a complicated ethical problem and has to be discussed on a social scale. Engineers in particular have to perceive their responsibility in this discussion. I hereby express my wish that this work shall not be used in systems aiming at localizing persons without their explicit consent.

## 1.2 Technical Context

The bigger part of this work was carried out in the context of an integration project with the title *A holistic approach towards the development of the first responder of the future* and the acronym E-SPONDER [VGC+10]. This project was funded by the European Commission within the 7<sup>th</sup> Framework Programme. E-SPONDER aimed to meet the demands of major crisis events. Namely, the lack of information regard-



ing the current location and health status of the first responders as well as secure and robust means of communication were to be addressed.

In the following, a list of the partners with their respective tasks within the project is given:

- *EXUS, Greece*, project coordination, back end software
- *University of Modena e Reggio Emilia, Italy*, network structure
- *CrisisPlan, Netherlands*, system specification, contact to end-users
- *CEREN, France*, system specification, contact to end-users
- *Immersion, France*, 3D user interface, visualization of first responder positions
- *Rose Vision, Spain*, standardization, optimization of crisis management operations
- *National Technical University of Athens, Greece*, communication infrastructure
- *CSEM, Switzerland*, biomedical sensors
- *Smartex, Italy*, integration of sensors into garment worn by first responders
- *Technische Universität Dresden, Germany*, local positioning system
- *YellowMap, Germany*, provide map material and point of interest handling for visualization
- *Panou, Greece*, development of the mobile operations center

In large scale crises, the exact location of any first responder is a crucial information in order to effectively manage the situation. The variety of different crisis scenarios and the high accuracy demands rule out the use of GNSSs as the main technology. A dedicated local positioning system (LPS) is necessary to meet all the demands.

The E-SPONDER system (see Fig. 1.1) consisted mainly of three components: the distant emergency operation center (EOC), the on-site mobile emergency operation centers (MEOCs) and the first responder units (FRUs). The communication among first responders (FRs) as well as between FRs and the MEOC or EOC were realized via a dedicated incident area network which had to be independent from existing infrastructure such as cellular networks, since those might be either damaged in a crisis event or may suffer from overload. Furthermore, the communications infrastructure was to be cheap, easily scalable and able to provide broadband data services for sensor data and even video. Last but not least, data security had to be realized in order to protect the health data of the first responders.

One integral part of the FRUs was a small module able to determine the respective FR's position. This position would then be transmitted via the incident area network to the MEOC and ultimately the EOC where the position of every FR would then be displayed within the surrounding. While in the MEOC, the position of each and every first responder is necessary, those points can be clustered in the EOC and thus facilitate tactical decisions based on real-time data.

As for every positioning system, the designed LPS relied on the position of reference stations (RSs) with known positions. For GNSS, these reference stations are satellites, for the LPS, several RSs have to be deployed around the incident area.

From the application in crisis environments, the following requirements for the LPS were derived:

- *3D Positioning* In complex indoor scenarios, it is important to determine the location of the first responders in three dimensions.
- *Robustness* Due to the variety of crisis scenarios, the LPS has to operate in many environmental conditions. In particular, multi path propagation has to be expected.
- *Real-time Capability* Since the movements of the first responders have to be tracked, the update rate of the LPS was specified to one second.
- *Scalability* The extent of every crisis is different and cannot be foreseen. Moreover, different crisis personnel may enter during the course of a disaster event. For this reason, a system had to be developed which can be scaled very easily. The system performance, in particular update rate and accuracy must not be influenced by the number of first responders. For this reason, a system similar to GPS was developed, where the mobile stations can determine their position without the need of a transmission back to the reference stations.
- *Coverage and Accuracy* A range of at least 300 m and an accuracy of 1 m was specified.
- *Easy-to-deploy Infrastructure* The reference stations of the LPS had to enable an easy setup. They were to be completely wireless. Once switched on, they should determine their own position relative to a global coordinate system and start positioning without any other user intervention.
- *Licence-free Operation* To comply with regulations in different regions of the world, the system has to operate in a license free Industrial, Scientific, Medical (ISM) band.

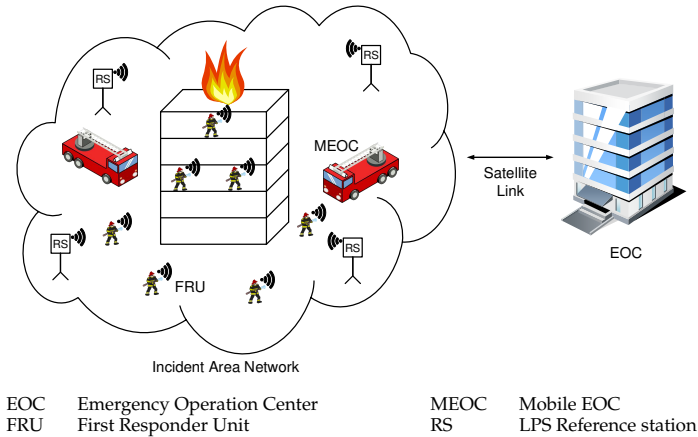


Figure 1.1: The E-SPONDER system

To meet all the above demands, a system was designed which relies on the well known FMCW radar principle. Unlike in classical radars, however, the radar targets (FRUs) were equipped with active tags in this case. Just like in GNSS systems, the RSs would then emit frequency chirps which are received in the FRUs. From the time difference between each frequency chirp received in the mobile tags, the position of the FRU itself can be calculated with respect to the reference stations. The calculated position is then fed back to the system using the communication infrastructure. Without the need of a wireless transmission back from each mobile tag to the reference stations, this system can support an unlimited number of mobile tags. The positioning principle used is called time difference of arrival (TDoA) and is explained in more detail in section 2.5 on page 18. The position of the reference stations is determined using GPS.

One of the main limitations resulting from the requirements is the restriction to the ISM band, which limits both the allowed transmission power and bandwidth. In FMCW radar, limiting the bandwidth results in a reduced resolution, limiting the transmission power reduces the system coverage. Within the E-SPONDER project, two strategies were researched to improve the resolution while complying with the ISM band restrictions. The first strategy was to use the information from two frequency bands by designing a dual-band frontend, this work is presented in [Jor15]. The second approach was to make use of antenna diversity by designing a beamsteering frontend for an FMCW radar. In order to be able to fuse both benefits, the beamsteering frontend was designed to be dual-band capable as well.

### 1.3 Objectives and Structure

In this work, a dual band, fully integrated RF beamsteering frontend for a secondary FMCW radar system was designed in an advanced semiconductor technology. First, the basics of FMCW radar systems are summarized. These basics are extended towards multipath propagation and RF beamsteering frontends. From these calculations, design specifications are derived.

The goal of this work was the development of a fully integrated dual-band RF beamsteering frontend for use with a secondary FMCW radar system. First, improved amplitude control circuits were designed, modeled and characterized. Using these, a vector modulator was designed and tested. An integrated RF beamsteering frontend was developed and fabricated. Last but not least, an RF beamsteering FMCW radar modules were assembled. They were configured in a master slave configuration using frequency chirps to measure their distance. They were characterized in different scenarios.

The focus of this work was the circuit design of the beamsteering frontend. For verification however, many blocks have been reused, namely the fully integrated FMCW radar system described in [Jor15] as well as signal processing hard- and software. These works have been discussed here only so far as necessary.

The thesis is structured as follows. Following this introduction, the fundamentals of FMCW radars are presented in chapter 2. The implications of multipath propagation on FMCW signals and the benefits of beamsteering frontends are presented. The chapter closes with a discussion on distance measurement error.

In chapter 3, the chosen technology will be introduced, followed by some basic metall-oxide-semiconductor field-effect transistor (MOSFET) device physics relations.

Chapter 4 discusses the design of amplitude control stages, which are essential building blocks for vector modulators. An attenuator and a variable gain amplifier (VGA) are presented and compared.

In chapter 5, the design of the vector modulator application specific integrated circuit (ASIC) is discussed with all its building blocks. The measured results of the fabricated ASIC are compared with other works.

Finally, chapter 6 is to document the design of the beamsteering FMCW radar system. The scenarios used to verify the system are described in detail, the results are presented and summarized. Chapter 7 summarizes the thesis and provides an outlook on possible future work.

## 2 Fundamentals

In this chapter I will discuss the fundamentals of FMCW radar based system as far as they are of concern for the current work. For questions beyond this short and rough introduction, I suggest to refer to the excellent work of [Jor15]. The system described by the author was adopted in this work and enhanced by a multi antenna frontend.

In section 2.2, the problems faced in indoor reflective environments are discussed with special regard to the FMCW principle.

Section 2.3 discusses the implications on RF combining multi antenna frontends and motivates the design goals for the blocks described in the later chapters.

Finally, section 2.4 discusses the distance measurement error and defines performance measures for the designed system.

### 2.1 FMCW Radar

The author of [Jor15] provides a thorough classification of LPS and distance measurement systems. He justifies the choice of the FMCW radar principle with its robustness and the high range in comparison with other principles. In Fig. 2.1a, a block diagram of a classical FMCW primary radar is shown. A chirp generator emits a chirp signal  $s_{\text{tx}}(t)$  which is sent towards the radar target or reflector. The backscattered chirp  $s_{\text{rx}}(t)$  is received by the radar and mixed down with  $s_{\text{tx}}(t)$  thus generating the baseband signal  $s_{\text{bb}}(t)$ . In Fig. 2.1b, the chirp signal is shown. In most cases, the generated chirp is a signal whose frequency increases linearly with time (solid line). The backscattered signal arrives at the radar after the time-of-flight  $\tau$  (dashed line). Multiplying both signals generates the baseband signal  $s_{\text{bb}}(t)$ . Since the chirp frequency increases linearly, the baseband signal is a single tone with the frequency  $f_{\text{D}}$ , the so called *beat frequency*. Since the chirp bandwidth  $B_{\text{fm}}$  and duration  $T_{\text{fm}}$  are known,  $\tau$  can be calculated from the beat frequency. The distance measurement thus comes down to a frequency measurement of  $f_{\text{D}}$ , this is performed in the signal processing block. In this particular system,  $s_{\text{tx}}(t)$  is sampled and a fast fourier transform (FFT) is performed. In the FFT result, a simple maximum detection can be performed to determine the beat frequency  $f_{\text{D}}$ .

The instantaneous frequency of  $s_{\text{tx}}(t)$  can be written as

$$\omega(t) = \omega_0 + \mu' t \quad (2.1)$$

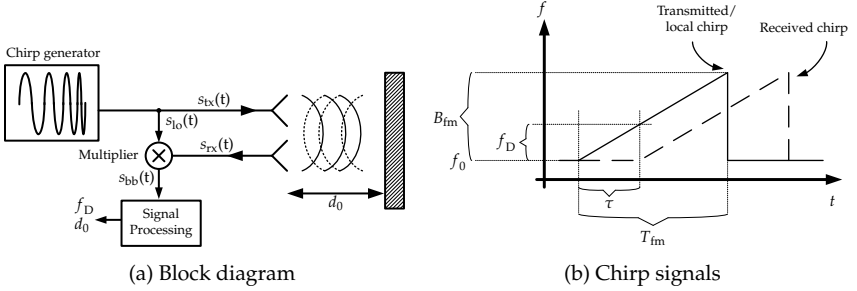


Figure 2.1: Block diagram and chirp signals for FMCW radar from [Jor15]

where  $\mu' = 2\pi \cdot B_{\text{fm}}/T_{\text{fm}}$  is called the *chirp gradient*. The frequency chirp can be described as

$$s_{\text{tx}}(t) = A_{\text{tx}} \cdot \cos\left(\omega_0 t + \frac{\mu'}{2} t^2 + \varphi_{\text{tx}}\right) \quad (2.2)$$

This signal is sent towards the reflector, gets reflected and is received at the radar after the time-of-flight  $\tau$  which is related to the distance  $d_0$  via the speed of light:

$$\tau = \frac{2 \cdot d_0}{c} \quad (2.3)$$

The received signal can then be expressed by

$$s_{\text{rx}}(t) = A_{\text{rx}} \cdot \cos\left(\omega_0(t - \tau) + \frac{\mu'}{2}(t - \tau)^2 + \varphi_{\text{rx}}\right) \quad (2.4)$$

By multiplying 2.2 with 2.4, the mixing process can be modeled, the resulting signal is labeled by  $s_{\text{filt}}(t)$  in Fig. 2.1a. Following the mixer, a low-pass filter eliminates the high frequency mixing product. The result is the single tone signal in the base band  $s_{\text{filt}}(t)$  and can be described by

$$s_{\text{filt}}(t) = A_{\text{filt}} \cdot \cos\left[\underbrace{\mu' \tau \cdot t + \tau \left(\omega_0 - \frac{\mu'}{2} \tau\right) + \varphi_{\text{tx}} + \varphi_{\text{rx}}}_{\varphi_{\text{filt}}}\right] \quad (2.5)$$

The term  $\omega_D = \mu' \tau$  is the angular beat frequency. Since the frequency ramp is of limited duration  $T_{\text{fm}}$ , equation 2.5 has to be multiplied by a rectangular window